Low-Complexity Channel Estimation for the Wireless OFDM Systems

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***Abstract*—Wireless orthogonal frequency division multiplexing (OFDM) systems need accurate channel estimation in order to compensate for the distortions caused by propagation through the dispersive channel. This work compares two fundamentally different pilot-assisted channel estimation algorithms: the maximum likelihood and the linear minimum mean squared error criteria based. Both performance and computational complexity are analysed to establish a feasible solution.**

***Index Terms*—channel estimation, multipath, OFDM, PSAM.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) technology has become increasingly popular nowadays. Contemporary wireless OFDM systems offer high transmission rates due to the use of spectrally efficient quadrature amplitude modulation (QAM). Coherent demodulation of the QAM signals requires explicit knowledge of the channel response, in order to minimise the probability of detection error [5]. Thus accurate channel estimation is of crucial importance to keep system performance at a proper level.

In most application scenarios of the wireless OFDM systems, the propagation channel can be assumed slow fading, which exhibits strong frequency correlation (within one OFDM symbol) and time correlation (across several symbols) properties. Priority should be addressed to the accurate frequency-domain estimation performed on the interval of one OFDM symbol as it gives better performance for a given complexity than the approaches exploiting long-term time correlation of the channel. Derivation of such a scheme is presented in [3], where it is referred to as the block-oriented linear minimum mean squared error (LMMSE) estimator. The advantage of the LMMSE method is that it belongs to the so-called non-parametric channel estimation techniques, which do not rely on a specific channel model.

Another approach in the OFDM channel estimation deals with parametric channel models. In [2] frequency correlation of the channel is expressed by the finite multipath delay spread. This property allows using deterministic model, parameterised by the channel impulse response, to derive maximum likelihood (ML) estimator. In [1] an alternative decision-directed implementation of the ML estimation algorithm is proposed. Being based on the same idea, it is free of some shortcomings of the previous scheme, namely a bound on the number of training subcarriers.

In this paper, we present a comparative analysis of the two fundamentally different low-complexity channel estimation techniques, which can be used in the wireless OFDM systems – low-rank LMMSE and ML. The objective is to examine a number of aspects: design limitations, complexity-performance comparison, robustness to changes in channel statistics, etc. The rest of the work is organised as follows. In Section II OFDM system model is described. Section III introduces the channel estimation algorithms. The complexity of these algorithms is analysed in Section IV. Section V contains performance evaluation results obtained by means of simulations. This is followed by conclusions in Section VI.

II. OFDM System Model

In the given work a single-input-single-output discrete-time baseband OFDM model is considered. It includes transmitter, receiver and equivalent bandlimited channel model (Fig. 1). The transmitter and the receiver are assumed to have ideal timing and frequency synchronisation.

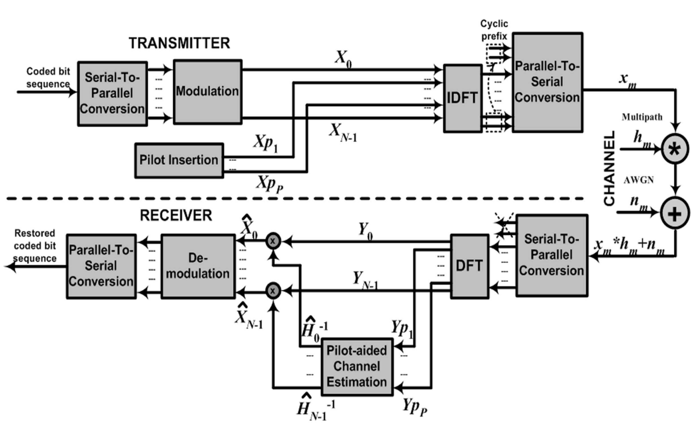

Fig. 1. Baseband pilot-assisted OFDM system

In the transmitter, serial stream of (coded) data bits is divided into N parallel binary streams, each of which passes through a linear modulation scheme. The ith OFDM symbol is formed as the result of the inverse discrete Fourier transform (IDFT), applied to N parallel complex-valued modulation subsymbols $X_n(i), \quad n = 0,...,N-1$. The resultant waveform is converted to a serial sequence of samples. Before transmission each OFDM symbol is prepended with the cyclic prefix, which is a copy of the last portion of the OFDM symbol.

In the considered scenario the channel is assumed to be slowly time-varying (or slow fading), i.e. the channel response is approximately constant during one OFDM symbol. However, if the channel is rapidly time-varying, typically in situations when the transmitter and the receiver are mobile relatively to each other or to the objects on the propagation path, the channel response undergoes strong changes on the interval of one OFDM symbol (fast fading), which lead to the loss of orthogonality between subcarriers and, as a consequence, severe signal distortions due to intercarrier interference (ICI). In [1] it is asserted that a time-varying channel can be well approximated by the time-invariant model during time interval T if

$$T \le 0.01 / f_{\mathrm{D}}, \tag{1}$$

where $f_{\mathrm{D}} = f_{\mathrm{c}} v / c$ is the maximum Doppler frequency, f_{c} is the RF carrier frequency, v is the speed of relative movement between the transmitter and the receiver, and c is the speed of light. Criterion (1) is usually satisfied for all fixed or slow-moving high-rate wireless OFDM systems, operating in the band 2-11 GHz, as the duration of the OFDM symbol is much shorter than the coherence time of the radio channel.

Assuming a multipath time-invariant channel, the length of the cyclic prefix N_{cp} can be selected big enough to accommodate finite channel impulse response $h_m(i), \quad m = 0,...,L$, where $h_m(i)$ could be modelled as the i.i.d. zero-mean complex Gaussian variables, with Rayleigh distribution of magnitudes and uniform distribution of phases, and the maximum sample-normalised delay spread $L \le N_{\mathrm{cp}}$. Thus, the intersymbol interference (ISI) between consecutive OFDM symbols will be eliminated. The unit-energy normalised power delay profile of the channel is typically assumed to be exponentially decaying, i.e.

$$\mathrm{E}\{|h_m(i)|^2\} = \frac{1 - \mathrm{e}^{-\alpha_0}}{1 - \mathrm{e}^{-(L+1)\alpha_0}} \mathrm{e}^{-\alpha_0 m}, \quad 0 \le m \le L, \tag{2}$$

where the exponential factor $\alpha_0 > 0$ is determined from the sample-normalised root-mean-squared delay spread τ_{RMS} solving numerically the nonlinear algebraic equation

$$\frac{1}{2(\cosh \alpha_0 - 1)} - \frac{(L+1)^2}{2[\cosh(L+1)\alpha_0 - 1]} = \tau_{\mathrm{RMS}}^2 \tag{3}$$

At the receiver side, after removing cyclic prefix and applying DFT to the ith OFDM symbol we get a vector of received data subsymbols [5]:

$$\mathbf{Y}(i) = [Y_0(i) \quad \cdots \quad Y_{N-1}(i)]^T = \mathbf{X}_{[D]}(i)\, \mathbf{H}(i) + \mathbf{N}(i), \tag{4}$$

where $\mathbf{X}_{[D]}(i)$ denotes a diagonal matrix with data symbols $X_n(i), \quad n = 0,...,N-1$, $\mathbf{H}(i) = [H_0(i) \quad \cdots \quad H_{N-1}(i)]^T$ is the DFT of the channel impulse response $h_m(i), \quad m = 0,...,N-1$, and $\mathbf{N}(i) = [N_0(i) \quad \cdots \quad N_{N-1}(i)]^T$ are the DFT-transformed white noise variables.

Before the parallel set of the received complex subsymbols Y_n can be demodulated, it is necessary to correct signal distortions caused by passing through the channel. As OFDM systems work by resolving the frequency domain, a simple block-oriented one-tap equaliser can be used. It divides the post-DFT received signal by the estimate of the channel frequency response (hereafter OFDM symbol index i is omitted for clarity, as the processing is done on a symbol-by-symbol basis, without considering correlation with the neighbouring symbols):

$$\hat{X}_n = Y_n / \hat{H}_n. \tag{5}$$

Channel estimates $\hat{H}_n$ can be obtained with the help of pilot-symbol assisted modulation (PSAM). This method relies on the transmission of the known training sequence on a small fraction of subcarriers, usually equally spaced over the whole band (Fig. 2).

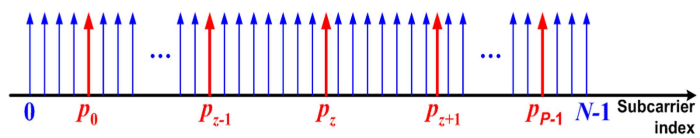

Fig. 2. OFDM spectrum with equally spaced pilot subcarriers

III. Channel Estimation

A. *Linear Interpolation of the LS Estimates*

In the simplest case the channel estimates are found by straightforward multiplying the received pilot subsymbols by the inverse of the reference pilot subsymbol values – the so-called frequency-domain least squares (LS) estimator, which can be written as

$$\hat{\mathbf{H}}_{\mathbf{P}}^{\mathrm{LS}} = \mathbf{X}_{\mathbf{P}[D]}^{-1} \mathbf{Y}_{\mathbf{P}} = \left[X_{p_0}^{-1} Y_{p_0} \quad \cdots \quad X_{p_{P-1}}^{-1} Y_{p_{P-1}}\right]^T, \tag{6}$$

where $\{p_0,..., p_{P-1}\}$ denotes the set of subcarriers, which are used to carry pilot subsymbols as shown in Fig. 2.

After that, the channel estimates $\hat{H}_{p_z}^{\mathrm{LS}}, z = 0,..., P-1$, obtained at the pilot positions, are interpolated over the whole band. Linear interpolation represents an example of a solution with the least possible computational complexity, when only one multiplication by a real factor is needed to compute the channel estimate for each data subcarrier:

$$\hat{H}_{p_z+k}^{\mathrm{LS}} = \hat{H}_{p_z}^{\mathrm{LS}} + \frac{k}{p_{z+1} - p_z}\left(\hat{H}_{p_{z+1}}^{\mathrm{LS}} - \hat{H}_{p_z}^{\mathrm{LS}}\right), \tag{7}$$

where $k = 1,..., p_{z+1} - p_z - 1$, and $p_{z+1} - p_z = \mathrm{const}$ $\forall z = 0,..., P-1$ for the case of the equally-spaced pilot subcarriers.

It is reasonable that the main disadvantage of the linear-interpolated LS estimator is its poor performance, as it takes into account neither statistical, nor structural properties of the channel. We include the LS scenario only for the purpose of illustrating the performance bound for the smallest complexity.

B. LMMSE Estimator

The linear minimum mean squared error (LMMSE) estimator, concerned in this paper, is designed to work in the frequency-domain only (one-dimensional). In particular, for the PSAM-based OFDM system, we describe an LMMSE pilot approximator that uses only P LS estimates as input values for the linear transformation:

$$\hat{\mathbf{H}}^{\mathbf{MMSE}} = \mathbf{Q}\mathbf{Y}_{\mathbf{P}}, \tag{8}$$

where the $N \times P$-size weighting matrix $\mathbf{Q}$ is selected in order to minimise MSE between the channel frequency response estimate $\hat{\mathbf{H}}^{\mathbf{MMSE}}$ and the assumed channel frequency response model $\mathbf{H}$ (described in terms of the second-order statistics):

$$J(\mathbf{Q}) = \frac{1}{N}\mathrm{E}\left[\left(\hat{\mathbf{H}}^{\mathbf{MMSE}} - \mathbf{H}\right)^H\left(\hat{\mathbf{H}}^{\mathbf{MMSE}} - \mathbf{H}\right)\right] =$$
$$= \frac{1}{N}\mathrm{E}\left[\left(\mathbf{Q}\mathbf{X}_{\mathbf{P}[D]}\mathbf{C}\mathbf{H} + \mathbf{Q}\mathbf{N}_{\mathbf{P}} - \mathbf{H}\right)^H\left(\mathbf{Q}\mathbf{X}_{\mathbf{P}[D]}\mathbf{C}\mathbf{H} + \mathbf{Q}\mathbf{N}_{\mathbf{P}} - \mathbf{H}\right)\right], \tag{9}$$

where $\mathbf{C}$ is the $P \times N$-size selection matrix with the elements $C_{m,n} = \begin{cases} 1, \text{ if } n = p_m \\ 0, \text{otherwise} \end{cases}$ that is needed to extract channel frequency response samples corresponding to the pilot subcarriers, and $\mathbf{N}_{\mathbf{P}} = \mathbf{C}\mathbf{N}$ denotes the noise affecting the received pilot subsymbols $\mathbf{Y}_{\mathbf{P}}$.

One can determine the optimal weighting matrix $\mathbf{Q}$ by minimising (9) with respect to $\mathbf{Q}$. Hence substitution of the corresponding result into (8) yields:

$$\hat{\mathbf{H}}^{\mathrm{MMSE}} = \mathbf{R}\mathbf{C}^H\left[\mathbf{C}\mathbf{R}\mathbf{C}^H + SNR^{-1}\mathbf{I}\right]^{-1}\mathbf{X}_{\mathbf{P}[D]}^{-1}\mathbf{Y}_{\mathbf{P}}, \tag{10}$$

where $\mathbf{R} = E\{\mathbf{H}\mathbf{H}^H\}$ is the autocorrelation matrix of the channel frequency response; $\mathbf{I}$ is the $P \times P$-size identity matrix; and SNR represents the signal-to-noise power ratio at the pilot subcarriers. Equation (10) is applicable to the case when pilot subsymbols transmitted on different subcarriers have equal constant power, i.e. $|X_{p_z}|^2 = |X_p|^2$.

The LMMSE estimator (10) uses *a priori* knowledge of the signal-to-noise ratio and the channel autocorrelation matrix $\mathbf{R}$, and is optimal when the statistical properties of the channel are known. Here the SNR value can be predefined: higher SNR ratios are preferable to obtain more accurate estimates. The robust estimator design necessitates account for the worst correlation of the multipath channel, namely when the channel power-delay profile is uniform. Under such an assumption the elements of the channel correlation matrix are expressed as:

$$R_{k,l} = \frac{1 - \exp\left[-\mathrm{j}2\pi(k-l)\left(N_{\mathrm{cp}}+1\right)/N\right]}{\mathrm{j}2\pi(k-l)\left(N_{\mathrm{cp}}+1\right)/N}, \quad k,l = 0,\ldots,N-1 \tag{11}$$

Straightforward product with the weighting matrix $\mathbf{Q}$ in the expression (10) involves NP complex multiplications that may represent a considerable computational load if P is large. In order to reduce it, [3] proposes to apply an optimal rank reduction for the matrix $\mathbf{Q} = \mathbf{R}\mathbf{C}^H\left[\mathbf{C}\mathbf{R}\mathbf{C}^H + SNR^{-1}\mathbf{I}\right]^{-1}\mathbf{X}_{\mathbf{P}[D]}^{-1}$, which is based on the singular value decomposition (SVD). This lets $\mathbf{Q}$ to be written as

$$\mathbf{Q} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^H, \tag{12}$$

where $\boldsymbol{\Lambda}$ is the $N \times P$-size diagonal matrix containing the singular values $\lambda_0 \ge \lambda_1 \ge \ldots \ge \lambda_{P-1}$, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices of the sizes $N \times N$ and $P \times P$ correspondingly, whose columns are the singular vectors. Interpreting $\boldsymbol{\Lambda}$ as a descending set of variances (powers) of the linear transform coefficients, one can exclude all but the r largest singular values $\lambda_0 \ge \ldots \ge \lambda_{r-1}$, i.e. $\boldsymbol{\Lambda}$ can be decomposed as $\boldsymbol{\Lambda} \approx \left|\begin{matrix} \boldsymbol{\Lambda}_{\mathbf{r}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{matrix}\right|_{N \times P}$, where $\boldsymbol{\Lambda}_{\mathbf{r}}$ is the $r \times r$-size diagonal matrix with elements $\lambda_0 \ge \ldots \ge \lambda_{r-1}$. Thus, the rank of the matrix $\mathbf{Q}$ is reduced from P to r ($r \le P$). In practice setting $r = L + \Delta = N_{\mathrm{cp}} + \Delta$, where Δ is selected according to Fig. 3, ensures accurate low-rank approximation without the error floor effect due to ignoring coefficients with smaller magnitudes ($\lambda_r \ge \ldots \ge \lambda_{P-1}$), and further increase of r does not give noticeable performance improvement as the remaining coefficient magnitudes are very close to zero. Dependence in Fig. 3 is obtained experimentally and describes an optimal Δ value choice for a given target SNR to achieve necessary precision level of the low-rank approximation. For example, in order to obtain approximated estimates with the relative error of MSE not exceeding 1% in respect to the full-rank LMMSE scheme Δ must be no less than 5 for target SNR ≤ 40 dB. There is, however, a restriction that for a realisable estimator construction $r = N_{\mathrm{cp}} + \Delta$, where $\Delta \ge 1$, must not exceed the number of pilot subcarriers P [4].

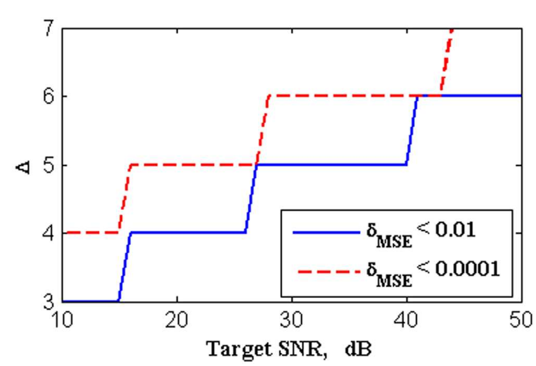

Fig. 3. Selection of Δ based on a target SNR

Notation (10) can be rewritten [7] as a combination of the orthogonal singular vectors $\mathbf{u}_i = \mathbf{U}^{<i>}$ and $\mathbf{v}_i = \mathbf{V}^{<i>}$ (where $(.)^{<.>}$ denotes a column of the matrix with the given index), then

$$\hat{\mathbf{H}}^{\mathrm{MMSE}} = \mathbf{U}\left|\begin{matrix} \boldsymbol{\Lambda}_{\mathbf{r}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{matrix}\right|\mathbf{V}^H\mathbf{Y}_{\mathbf{p}} = \sum_{i=0}^{r-1}(\lambda_i\mathbf{u}_i)\left\langle \mathbf{v}_i^*, \mathbf{Y}_{\mathbf{p}} \right\rangle, \tag{13}$$

where $\lambda_i\mathbf{u}_i$ is the scalar-vector product, and $\left\langle \mathbf{v}_i^*, \mathbf{Y}_{\mathbf{p}} \right\rangle$ is the inner Euclidean product of the two vectors. Note that the vectors $\lambda_i \mathbf{u}_i$ and $\mathbf{v}_i$ are pre-computed at the design stage from SVD of $\mathbf{Q}$, based on the preset target SNR and the channel correlation matrix, and stay fixed during the estimator's operation. The functional diagram of the low-rank pilot-assisted LMMSE channel estimator (13) is depicted in Fig. 4. One can see that the received pilot subsymbols are projected onto a (smaller) subspace spanned by the vectors $\mathbf{v}_i^*$, where estimation is performed by the singular value weighting. The final channel estimates are then found by linear combination of the basis vectors $\mathbf{u}_i$.

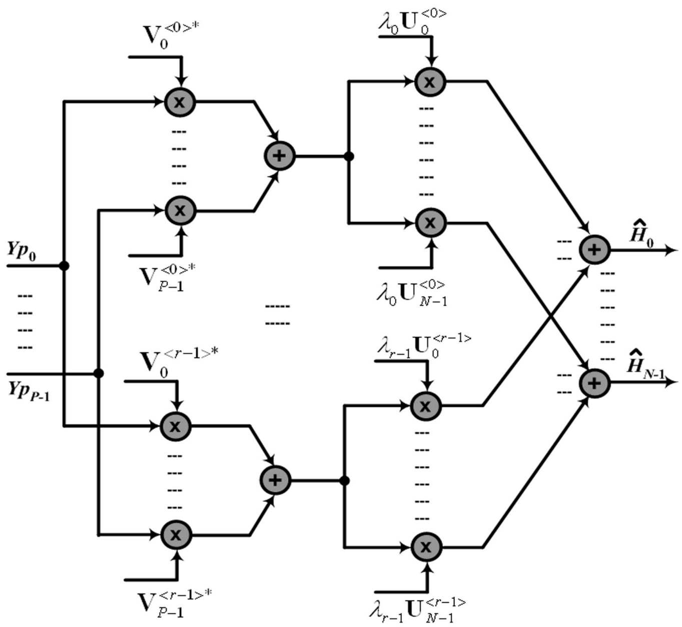

Fig. 4. Block diagram of the reduced-rank PSAM-driven LMMSE channel estimator

C. ML Estimator

An alternative in the channel estimator choice is the maximum likelihood (ML) criterion based approach [1]. The main idea is to obtain an estimate of the channel impulse response $\mathbf{h} = [h_0 \ \cdots \ h_L]^T$ that minimises the Euclidean distance function

$$\mathrm{d}(\mathbf{h}) = (\mathbf{Y_P} - \mathbf{X}_{\mathbf{P}[D]}\mathbf{H_P})^H (\mathbf{Y_P} - \mathbf{X}_{\mathbf{P}[D]}\mathbf{H_P}) = \\ = (\mathbf{Y_P} - \mathbf{X}_{\mathbf{P}[D]}\mathbf{CFBh})^H (\mathbf{Y_P} - \mathbf{X}_{\mathbf{P}[D]}\mathbf{CFBh}), \quad (14)$$

where $\mathbf{C}$ has been defined in the previous subsection, and $\mathbf{F}$ is the $N \times N$-size Fourier matrix with the elements $F_{m,n} = \exp(-\mathrm{j}2\pi mn/N)$. As the assumed channel impulse response model is constrained by only L+1 components, vector $\mathbf{h}$ of the size $(L+1)\times 1$ has to be zero-padded up to the size $N \times 1$ before DFT of $\mathbf{h}$ can be taken to yield the channel transfer function $\mathbf{H}$. For that reason the $N \times (L+1)$-size padding matrix $\mathbf{B}$ is used having the concatenated structure of $\mathbf{B} = \begin{vmatrix} \mathbf{I}_{(L+1)\times(L+1)} \\ \mathbf{0}_{(N-L-1)\times(L+1)} \end{vmatrix}$.

The ML estimation algorithm exploits the deterministic property of the limitedness of the channel impulse response, when the largest sample-normalised delay spread L is assumed to be less or equal to the length of the cyclic prefix N_{cp}, i.e. $L \le N_{\mathrm{cp}}$. This structural feature is closely linked with the frequency correlation of the channel, and if known precisely allows to construct an optimal estimator without any other knowledge of the channel [2].

Performing minimisation of (14) with respect to $\mathbf{h}$ leads to the following expression of the algorithm's output:

$$\hat{\mathbf{H}}^{\mathrm{ML}} = \mathbf{FB}(\mathbf{B}^H\mathbf{F}^H\mathbf{C}^H\mathbf{X}_{\mathbf{P}[D]}^H\mathbf{X}_{\mathbf{P}[D]}\mathbf{CFB})^{-1}\mathbf{B}^H\mathbf{F}^H\mathbf{C}^H\mathbf{X}_{\mathbf{P}[D]}^H\mathbf{Y_P}, \quad (15)$$

where the number of pilot subcarriers P is required to be no less than the channel impulse response length N_{cp}+1 for the matrix $\mathbf{W} = (\mathbf{B}^H\mathbf{F}^H\mathbf{C}^H\mathbf{X}_{\mathbf{P}[D]}^H\mathbf{X}_{\mathbf{P}[D]}\mathbf{CFB})^{-1}$ to be invertible.

A flow chart visualizing the described ML algorithm is shown in Fig. 5. The overall algorithm procedure represents a translation from the frequency domain to the time-domain and back using a Discrete Fourier Transform (DFT/IDFT) set. The actual estimation is performed in the time domain, where the number of parameters ($\hat{h}_0,...,\hat{h}_{N_{\mathrm{cp}}}$) is substantially smaller than in the frequency domain ($\hat{H}_0,...,\hat{H}_{N-1}$). This comes to a product in (15) with the weighting matrix $\mathbf{BWB}^H$, where the inverse term $\mathbf{W}$ has a small dimension and can be pre-calculated and stored as a set of the weight coefficients.

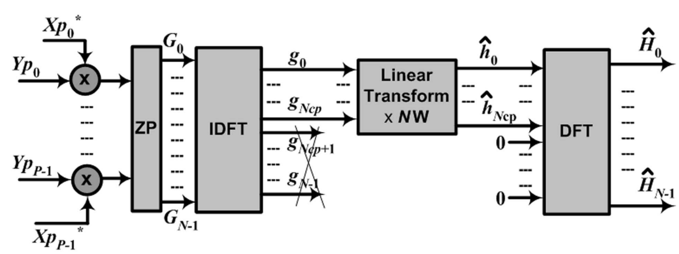

Fig. 5. Block diagram of the DFT-based ML estimator

IV. Algorithm Complexity

In this section the above channel estimation algorithms are analysed from the standpoint of computational complexity, which is traditionally expressed as a number of complex multiplications (*CMs*) required to obtain an estimate of the channel transfer function on the interval of one OFDM symbol.

A. Low-rank LMMSE Estimator

The form of the estimator given in (13) involves P CMs to compute the inner product $\langle \mathbf{v}_i^*, \mathbf{Y_p} \rangle$ for each vector $\mathbf{v}_i$. The resultant value is multiplied with the corresponding vector $\lambda_i \mathbf{u}_i$, representing $(N-P)$ CMs needed to obtain estimates for the $(N-P)$ subcarriers transmitting data. As the final estimates are computed by a sum of r vectors, the total number of CMs per OFDM symbol becomes $r(N-P+P) = rN$.

Assuming that the rank of the LMMSE approximator is equal to $r = L + \Delta = N_{\text{cp}} + \Delta$, where $\Delta = 5$, the complexity is expressed as $(N_{\text{cp}} + 5)N$ CMs per symbol.

B. ML Estimator

The time-frequency interpretation based on the pair of the fast Fourier transforms significantly diminishes complexity of the ML estimation algorithm. The conventional radix-4 implementation of the FFT/IFFT, which forms the base of most contemporary OFDM systems and requires approximately $3N/4(\log_4 N - 1)$ CMs, can be efficiently used for the translation of the P input values $Y_{p_z} X^*_{p_z}$, zero-padded up to N, to the time domain and converting back to the frequency domain at the final processing stage (Fig. 5). In a general case of non-uniformly spaced pilot subcarriers, a product of an IFFT-resultant complex vector with the hermitean positive-definite weighting matrix $\mathbf{W}$ having real elements on the main diagonal requires $(N_{\text{cp}} + 1)(N_{\text{cp}} + 1/2)$ CMs. For an equally spaced pilot arrangement $\mathbf{W}$ becomes a diagonal matrix, so that computational load is reduced to only $(N_{\text{cp}} + 1)/2$ CMs. Thus, the total CM amount of the ML estimator based on the conventional radix-4 FFT engines is calculated as $3N/2(\log_4 N - 1) + (N_{\text{cp}} + 1)(N_{\text{cp}} + 1/2) + P$ for the case of non-uniformly distributed subcarriers, and as $3N/2(\log_4 N - 1) + (N_{\text{cp}} + 1)/2 + P$ for the uniform pilot pattern. In [2] it is also reported that complexity can be lowered if using comb-optimised inverse FFT engine for the uniform pilot arrangement. It needs only about $N/4 + \log_2(N_{\text{cp}} + 1) - (N_{\text{cp}} + 1)/2$ CMs and represents a large gain for the big number of subcarriers N.

The graphs illustrating complexity of the estimators are shown in Fig. 6. The last term P in the total CM number expressions for the ML algorithm is set equal to the minimum allowed value of $P = N_{\text{cp}} + 1$. Indeed, its contribution is negligible as the number of pilots is several orders of the magnitude smaller than the total number of CMs required by the estimator. Therefore varying the number of pilot subcarriers does not substantially affect the computational complexity.

One can see that the FFT-based ML estimation has lower complexity than the low-rank LMMSE scheme. The increase of the length of the cyclic prefix, required to accommodate ISI due to more extensive channel dispersion, in most cases leads to even greater (up to an order of magnitude) difference between the ML and the LMMSE estimator. Complexity of the ML estimator approaches the one of the low-rank LMMSE only at very large N_{cp} values close to the OFDM symbol length, but this case is not of practical interest, as the maximum delay spread typically constitutes only a small percent of the total symbol duration. It should be noted that for a big number of subcarriers in the OFDM spectrum with equally spaced pilots, the quantity of CMs required by the ML algorithm is almost independent of N_{cp}, unlike the LMMSE case, in which it grows proportionally to the length of the channel impulse response.

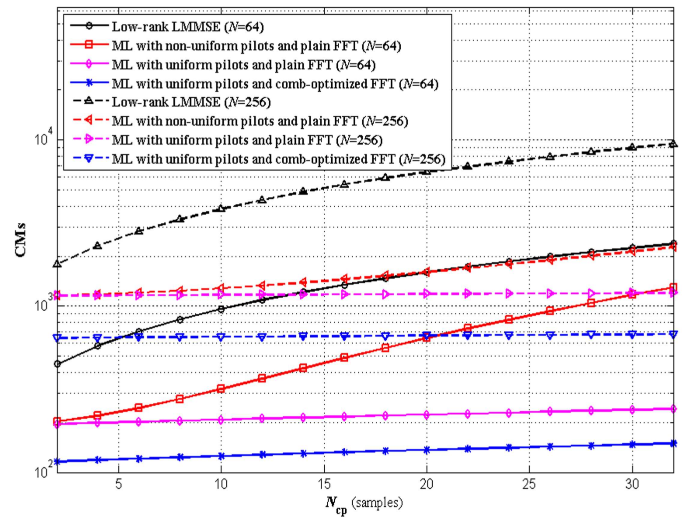

Fig. 6. Computational complexity of the channel estimation algorithms

V. Performance Analysis

In the following section we discuss the setup and performance evaluation of the simulated OFDM system, with different channel estimators.

A. Performance over the Slow Fading Multipath Channel

The simulation scenario consists of a PSAM-based OFDM system with 256 subcarriers and equal spacing of pilots (16 and 32 pilot cases are considered). Subcarriers transmitting uncoded data are modulated by the Gray-mapped QAM-16. The power of pilot subcarriers is constant and equal to the maximum subsymbol power. The length of the cyclic prefix is 7 samples. We choose the 8-tap bandlimited channel model ($L = 7$), statistically characterised as a wide sense stationary uncorrelated scattering (WSSUS) process [6], with the exponentially decaying power-delay profile and the sample-normalised delay spread equal to $\tau_{\text{RMS}} = 2$. The largest Doppler frequency is modelled according to the equation $f_{\text{D}} = 0.001/T$, where T is the OFDM symbol duration (including cyclic prefix). The target E_b/N_0 ratio for the LMMSE estimator was set to 30 dB, and the design was made for the channel with the uniform power-delay profile with the maximum delay spread equal to $N_{\text{cp}} + 1 = 8$. The rank of the LMMSE estimator was reduced to $r = N_{\text{cp}} + 5 = 13$.

Simulation results are shown in Fig. 7. Note that here we use the energy-per-bit-to-noise-ratio metric, which is common for performance analysis of the digital communication systems, and is linked to SNR of the PSAM-driven OFDM transmissions by equation [6]

$$\frac{E_{\text{b}}}{N_0} = SNR\frac{B}{R} = SNR\frac{BT}{k(N-P)} = SNR\frac{N+N_{\text{cp}}}{k(N-P)}, \qquad (16)$$

where $B = 1/T_s$ is the bandwidth of the system, $T = (N + N_{cp})T_s$ is the OFDM symbol period, and k is the number of bits carried by one modulation symbol ($k = 4$ for QAM-16).

The ML estimator exhibits better performance than low-rank LMMSE for both 16 and 32 pilot subcarriers. The difference between the 16-pilot ML estimator and the case when channel response is ideally known constitutes 1.5 dB on average. Note that the number of pilots does not substantially impact performance of the ML estimator. This feature plus lower complexity of the ML algorithm makes it attractive for the cases when OFDM system, driven by a small number of pilots, has to operate over dispersive channels with large maximum delay spreads. For the low-rank LMMSE algorithm, bigger amount of pilot subcarriers ($P = 32$) improves accuracy at higher E_b/N_0 values, so that BER converges with the ML solution.

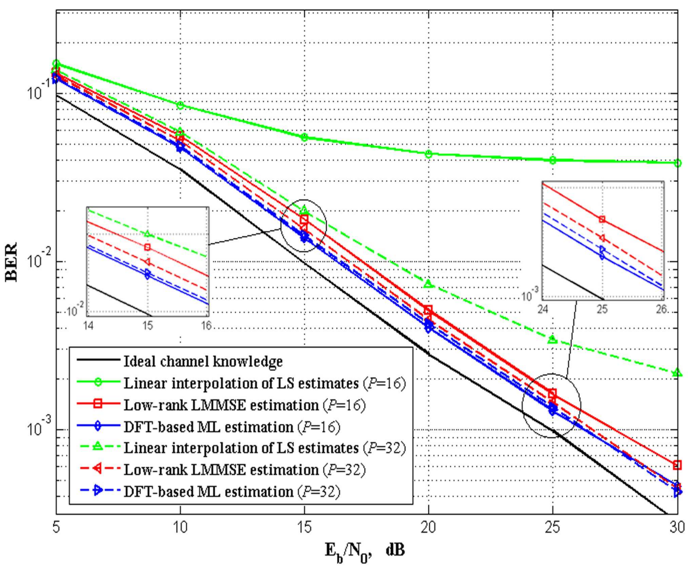

Fig. 7. Performance of different channel estimation algorithms

B. Effect of time variation of the channel

In the previous simulation scenario channel was modelled as effectively slow fading with the Doppler spread $f_D = 0.001/T$, i.e. by order of a magnitude smaller than required by (1) that makes its properties almost the same as of the time-invariant channel. An essential question is how channel estimators behave if time variation of the channel becomes stronger. To answer it we simulate OFDM system with the same setup over the same channel but with $f_D = 0.01/T$. Performance evaluation results are presented in Fig. 8.

Note that ICI, resultant from the loss of orthogonality of subcarriers due to their Doppler shifts, leads to irreducible error floor for all the considered channel estimation schemes. It is manifested by greater performance difference in comparison with the equaliser using ideal knowledge of the channel frequency response (e.g., approximately 2.7 dB at BER = 10^{-3} for the ML estimator). Subject to ICI distortion, estimates of the channel transfer function on the pilot subcarriers become more erroneous, so that performance difference of the low-rank LMMSE estimator with larger and smaller number of pilots diminishes.

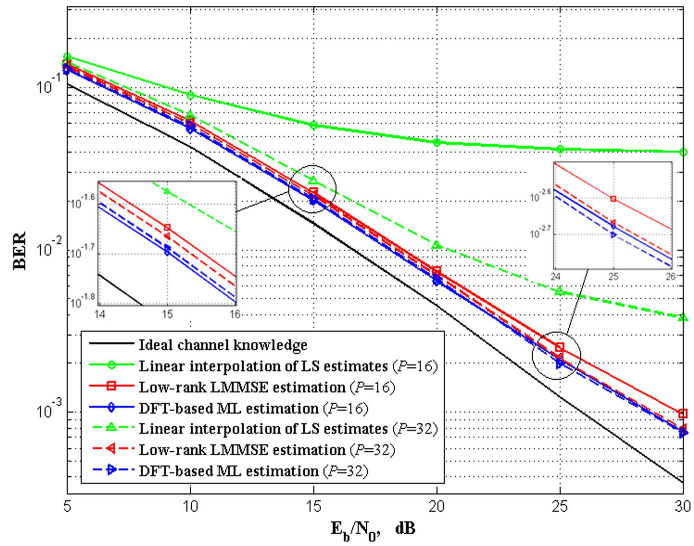

Fig. 8. Performance over the channel with faster time variation

VI. Conclusions

In this work two fundamentally different OFDM channel estimation algorithms have been compared. One of them (LMMSE) takes into account statistical properties of the channel, while the second (ML) handles deterministic channel features. Both complexity and the number of parameters to be known by the ML algorithm are much less than in case of the low-rank LMMSE estimator. The ML estimator outperforms the low-rank LMMSE, and represents a good solution for OFDM systems operating in the slow fading conditions. In the channel with more intensive time variations additional ICI compensation methods should be used to keep performance at the acceptable level.

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