Throughput Capacity of Wireless Ad Hoc Networks with Multicast Traffic

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***Abstract*— The focus of this paper is on presenting new results on throughput capacity of wireless ad hoc networks with multicast traffic. In seminal works [1], Gupta and Kumar introduced a new line of research. It is about the asymptotic throughput capacity of dense wireless ad hoc networks as a function of the number of nodes in the network. First, we present results on asymptotic behavior of a random ad hoc network based on simulations and analytical methods. Then, we present an upper bound on the throughput capacity of an ad hoc network with multicast traffic using a hierarchical routing strategy. We start by generating the upper bound and the strategy gain for the 2-level hierarchical strategy. Then, we generalize the result for multi-level hierarchical routing by giving the recurrence expression of the multicast gain. Finally, we apply the basic result to a broadcast traffic in the network.**

I. Introduction

Wireless ad hoc networks consist of a collection of nodes which communicate between them through a wireless channel and cooperate to route the information from a source node to its destination. Formally, a Mobile Ad hoc NETwork (MANET) is a system of wireless mobile nodes that dynamically self-organize in arbitrary and temporary network topologies. The principle characteristics of a wireless ad hoc network are its dynamic topology, limited bandwidth, energetic constraints, security problems and absence of infrastructure. There has been recent interest in designing and analyzing ad-hoc wireless networks since they could be an alternate wireless network architecture to the traditional hierarchical cellular architecture. The routing problem was the most studied until recently and many algorithms have been developed [2], [3].

In other words, in ad hoc wireless networks the nodes act both as sources of information as well as relays for traffic handling on behalf of other nodes through multihopping. Consequently, the simultaneous transmissions in ad hoc networks limit its per-user rate. So it is imperative to understand the fundamental capacity performance limits, in terms of throughput and delay, of ad hoc wireless networks, with the goal of designing resource allocation (power control, medium access, routing...) algorithms that allow to reach these performance limits. A new line of research has been initiated which is the asymptotic throughput of dense wireless networks. It has been established as a function of the number of nodes in the network. In seminal works [1], Gupta and Kumar showed that the per-user rate asymptotically decreases to zero when the number of nodes goes to infinity. It is then possible to achieve a per node capacity of $\Theta(\frac{1}{\sqrt{n \log n}})^1$, using global scheduling and near straight route lines. The $\log n$ factor is present because each node radio transmission range needs to increase as $\log n$ in order for an ad hoc network to stay connected with high probability as the number of nodes increases. In [4], Grossglauser and Tse have shown that if the nodes of the networks are moving quickly and independently, then a constant rate per communication pair can be achieved by a single relay strategy. However, this strategy can induce large delays, particularly in the situation where nodes are less mobile. In [5], El gamal and al. analyze the capacity/delay tradeoff by designing new communication strategies. In [6], the authors discuss the limitations of the work in [1], by taking a network information theoretic approach. The authors discuss how several co-operative strategies such as interference cancellation, network coding etc. could be used to improve the throughput. However these tools cannot be exploited fully with the current technology, which relies on point-to-point coding, and treats all forms of interference as noise.
In this paper, we develop performance bounds on the throughput capacity of an ad hoc network with multicast traffic.

II. System Model

A. Network Model

Let n nodes be uniformly and independently distributed in a planar square of unit area. Two nodes can directly communicate with each other if the distance between them is no more than $r(n)$, where $r(n)$ is the signal range of these nodes. The ad hoc network consists then of n nodes X_i, $i \in [1..n]$, each node can be either a source, a destination or a relay node. Furthermore, we assume that each source node has an infinite reservoir of packets [2] to send to its destination. We will denote by d_{ij} the distance between nodes i and j. Finally,

[1] $f(n) = \Theta(g(n))$ if and only if

$$|f(n)| \leq c|g(n)| \quad \text{and} \quad |g(n)| \leq c'|f(n)|$$

for constant c and c' and for a large enough n

[2] This implies that we neglect buffering problems

each node can transmit at W bits per second over a common wireless channel.

B. Interference Model

We consider an ad hoc network with n nodes which share a common wireless channel and can act as transmitters and receivers. Assume time is divided into equal slots. In each time slot, a node is scheduled to send data. A node cannot transmit and receive data simultaneously and a node can only receive data from another node at the same time.

For the interference model, we adopt the *"the protocol model"* presented in [1]. Suppose node X_i transmits to a node X_j. Then this transmission is successfully received by node X_j if and only if:

- The distance between X_i and X_j is no more than $r(n)$, i.e.,
$$|X_i - X_j| \leq r(n)$$
- For every other node simultaneously transmitting over the same channel
$$|X_k - X_j| \geq (1 + \Delta)r(n)$$

The quantity $\Delta > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting on the same channel at the same time. It also allows for imprecision in the achieved range of transmissions.

III. Technical Lemmas

In this section, we present results on the asymptotic behavior of random ad hoc netwoks. Based on simulations of the network topology and the traffic model we show numerically the validity of some technical lemmas [1], [4], [5].

A. Cell partitioning

As illustrated in figure 1, we assume the area of the network to be partitioned in a set of k regular cells. Each cell is a square of area $a = 1/k$. The number of cells will in general depend on n, hence we will use $k(n)$ to represent this parameter.

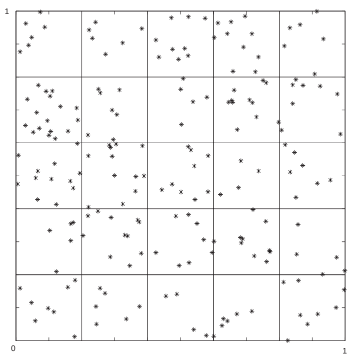

Fig. 1. Unit Square Network with cell partition, $n = 200$

B. Node distribution properties

The nodes are uniformly and independently distributed in a planar square of unit area

Lemma 3.1: For a suitable choice of $k(n)$, no cell is empty with high probability as n becomes large.

Proof: For a network of unit area with n nodes uniformly and independently distributed and $k(n)$ cells of $a(n)$ area, the probability that a cell is empty is equal to $(1 - a(n))^n$. By using the union bound, we have:

$$\begin{aligned} Pr[\text{at least one cell is empty}] &\leq k(n)((1 - a(n))^n \\ &= k(n)(1 - 1/k(n))^n \\ &\leq k(n)\exp\left(-n/k(n)\right) \end{aligned}$$

where we used the fact that $k(n) = 1/a(n)$ and $(1-x) \leq e^{-x}$ Following results on occupancy problems [7], we set $k(n) = c\frac{n}{\log n}$, $c \geq 1$ and obtain:

$$Pr[\text{at least one cell is empty}] \leq c\frac{1}{\log(n)n^{c-1}}$$

which goes to zero as n increases indefinitely. ■

By using simulations, the variation of the probability of finding an empty cell as the number of nodes increases in the network is represented in figure 2, assuming $c = 1$. It is indeed confirmed that the lemma result holds.

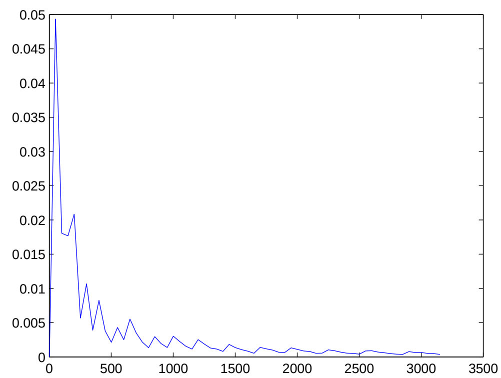

Fig. 2. Probability of finding an empty cell versus the number of nodes n

Let b_j be the number of nodes in the cell C_j, $1 \leq j \leq k(n)$ and $E = \left\{\frac{1}{2}\frac{n}{k(n)} \leq b_j \leq 2\frac{n}{k(n)} \forall j\right\}$

Lemma 3.2: For a suitable choice of $k(n)$, $\lim_{n\to\infty} Pr[E] = 1$

Proof: b_j is a binomial random variable, with expectation $\frac{n}{k(n)}$. Applying the Chernoff bounds we obtain:

$$Pr[b_j \leq \frac{1}{2}\frac{n}{k(n)}] \leq \exp -\frac{n}{8k(n)}$$

and

$$Pr[b_j \geq 2\frac{n}{k(n)}] \leq \exp\left(-f(1)\frac{n}{k(n)}\right)$$

where $f(x) = (1 + x)log(1 + x) - x$.

$$Pr\left[\frac{1}{2}\frac{n}{k(n)} \le b_j \le 2\frac{n}{k(n)}\right] = 1 - Pr\left[\left(\frac{1}{2}\frac{n}{k(n)} \le b_j \le 2\frac{n}{k(n)}\right)^c\right]$$
$$= 1 - Pr\left(b_j \le \frac{1}{2}\frac{n}{k(n)}\right) \cup \left(b_j \ge 2\frac{n}{k(n)}\right)$$

Using the union bound we obtain then:

$$Pr[E] \ge 1 - \left(\exp\left(-\frac{n}{8k(n)}\right) + \exp\left(-f(1)\frac{n}{k(n)}\right)\right)$$
$$\ge 1 - 2\exp\left(-\frac{n}{8k(n)}\right)$$

Similarly to the last lemma, we set $k(n) = c\frac{n}{\log n}$, $c \ge 1$ and obtain:

$$Pr[E] \ge 1 - \frac{2}{n^{8c}}$$
$$\Rightarrow \lim_{n\to\infty} Pr[E] = 1$$

■

C. Bound on the number of lines through a cell

Considering the uniform traffic model [1], each source chooses uniformly a destination from the rest of the nodes in the network. In this model, the authors showed that the mean number of Source-Destination (S-D) lines passing through a cell is $O(\sqrt{n\log n})$. In this part, we aim to confirm the analytical result by using a numerical method based on simulations for the network model considered in this paper.
Considering the ad hoc network of square area and partitioned in $k(n) = \frac{n}{\log n}$ cells as described before, we evaluate numerically the number of lines passing through the central cell. In fact, we can remark intuitively that the number of S-D lines through the cell in the center of the network area is more likely to be greater than the number of lines going through another cell. By simulation of the uniform traffic in the network, we can find the behavior of the average number of routes passing through the central cell with respect to the number of nodes n. It is found [fig.3] that the ratio of the measured number of lines passing through the cell in the center and the expected one is constant as n becomes large:

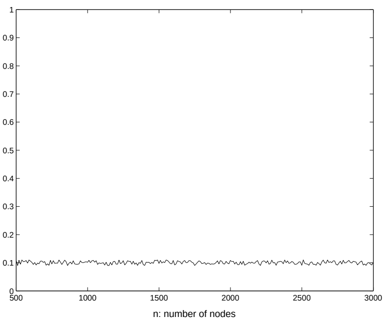

Fig. 3. Measured divided by Expected

Then from the simulations, we have:

$$\mathbb{E}[\text{N=Number of lines crossing central cell } C] \sim \alpha\sqrt{n\log n} \quad (1)$$

where α is a positive constant ($\alpha \sim 0.1$ by simulations). Let's consider the Binomial random variables X_i defined as follow:

$$X_i = \begin{cases} 1 & \text{if the S-D line } L_i \text{ cross center cell C} \\ 0 & \text{else} \end{cases}$$

Let $p = Pr[X_i = 1]$ then, as $N = \sum_i X_i$ is a Bernoulli random variable:

$$\mathbb{E}[N] = np \quad (2)$$

$$\mathbb{V}ar[N] = np(1-p) \quad (3)$$

Using the simulations results, we obtain:

$$np = \alpha\sqrt{n\log n}$$

Using Tchebycheff inequality, we have

$$Pr\left(N \ge 2\mathbb{E}[N]\right) = Pr\left(N - \mathbb{E}[N] \ge \mathbb{E}[N]\right)$$
$$\le Pr\left([N - \mathbb{E}[N]]^2 \ge \mathbb{E}^2[N]\right)$$
$$\le \frac{\mathbb{E}\left[[N-\mathbb{E}[N]]^2\right]}{\mathbb{E}^2[N]}$$
$$= \frac{\mathbb{V}ar[N]}{\mathbb{E}^2[N]}$$

By using (2) and (3), we obtain:

$$Pr\left(N \ge 2\mathbb{E}[N]\right) \le \frac{np(1-p)}{np^2}$$
$$\le \frac{1}{\alpha\sqrt{n\log n}} \quad \to 0 \quad \text{as } n \text{ becomes large}$$

Then, we showed that the number of source-destination lines going through a cell in the network scales as $O\left(\sqrt{n\log n}\right)$.

D. The average neighbors number scaling law

Similarly to the section before, we consider the network with n nodes uniformly and independently distributed in the unit square. Following the result of Gupta and Kumar on the asymptotic connectivity of the network [8], we assume that two node are connected if their distance is smaller than the critical radius for connectivity, $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$. Then the average number of neighbors for each node is $O(\log n)$ as n becomes large [1].
In the figure 4, we represent the variation of the ratio of the average number of neighbors for all the nodes and the expected one, $\log n$. The simulations results show that the variation of this ratio is constant (around 3). Then, following the same probabilistic method used in the section (III-C), we can achieve this numerical "proof" and confirm that the average number of neighbors of the nodes in the random network scales as $O(\log n)$.

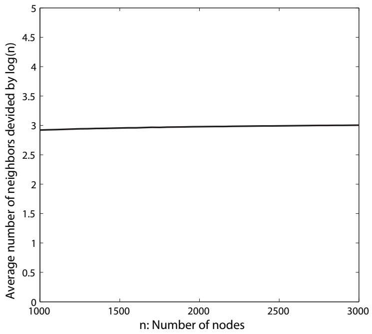

Fig. 4. Measured divided by Expected

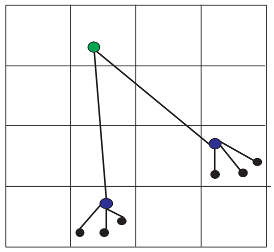

Fig. 5. Multicast Hierarchical Routing

IV. The Multicast Traffic

A. Motivation

Although the model developed by Gupta and Kumar [1] initiated the majority of research activities in this field, their model of point-to-point traffic presents limitations in several applications [9], [10]. Indeed, for voice communications between two terminals, the model presented is adequate. However, we find situations where some nodes support more traffic than others. Let us think of a sensor network where various sensor nodes collect information and send them to a principal node which handles all information received. In this case, the uniform model of point-to-point traffic becomes limited. Moreover, in several applications such as the Internet, a multicast traffic is in effect, where a source sends data to a certain number of destination nodes.

Multicast routing protocols are designed and deployed to reduce communication costs when dealing with applications involving communications between multiple users. In a multicast session, one of several sources transmits the same data to multiple destinations [11], [12]. Multicast routing protocols are in charge of building optimal paths to reach all destinations.

In this work, we develop performance bounds on the throughput capacity of a network with multicast traffic.

B. Routing Strategy

We adopt a hierarchical routing strategy. In fact, we consider a source node which sends the same data to m destination nodes independently and randomly chosen. The unit square is divided into k square cells (convex clusters) of the same area ($1/k$). The convexity of the clusters insure that the traffic between the nodes in a cell remains inside the cell and doesn't involve intermediate nodes from other clusters (cells). As a first attempt we will consider a two-level hierarchical routing [fig.5]. The source sends the same data to the cluster heads in each cell containing at least one of the m destinations. Then, each cluster head will route the packets to the destinations in its cell. The advantage of this strategy is in the shared path gain of the multicast tree. Assuming the bandwidth is divided among different levels, we identify two cases. On the one hand the high density multicast (large number of destination nodes) and on the other hand, the low density multicast (small number of destination nodes).

Lemma 4.1: For the high density multicast case, there is, with high probability, at least one destination in each cluster and on average m/k destination nodes in each of them.

Proof: Let X_i^k a random variable:

$$X_i^k = \begin{cases} 1 & \text{if the node } i \text{ is in the cell } k \\ 0 & \text{else} \end{cases}$$

While the destination nodes are uniformly and independently distributed, the probability that a node i is in the cluster k is $1/k$. Then $E[X_i^k] = 1/k$. We define the random variable $M_k = \sum_{i=1}^m X_i^k$ which represents the number of destination nodes in the cell k. Since $\left\{X_i^k\right\}_1^n$ is a sequence of i.i.d random variables with $E[X_i^k] = 1/k$, using the law of large numbers, we obtain with high probability:

$$\frac{M_k}{m} = \frac{1}{m}\sum_{i=1}^{m} X_i^k \to \frac{1}{k} \textit{ when } m \to \infty$$

Since $k = o(m)$, then $\lim_{m\to\infty} \frac{m}{k} \to \infty$. Finally, $M_k = \frac{m}{k} \to_{m\to\infty} \infty$.

■

Consequently, the source sends the data packets to the k cluster heads (the choice of the cluster head is not important) during the first level of the hierarchy routing.

Lemma 4.2: For the low density multicast case, only a proportion k' of the clusters are involved in the multicast communication, $k' = k - k(1-\frac{1}{k})^m$.

Proof: Let X_l be a random variable defined us follow:

$$X_l = \begin{cases} 1 & \text{if the cluster } l \text{ contains at least one destination} \\ 0 & \text{else} \end{cases}$$

$$\begin{aligned} Pr[X_l = 1] &= 1 - Pr[X_l = 0] \\ &= 1 - (1-\frac{1}{k})^m \end{aligned}$$

Then,

$$E[X_l] = 1 - (1-\frac{1}{k})^m$$

if we define $Y_k = \sum_{l=1}^k X_l$, we obtain that the average number of clusters where there is at least a destination node is $E[Y_k]$:

$$E[Y_k] = \sum_{l=1}^{k} E[X_l] = k - k(1-\frac{1}{k})^m$$

Then $k' = k - k(1-\frac{1}{k})^m$ ■

Consequently, in this case the source node sends data packets to $k' < k$ cluster heads.

V. Capacity of Ad Hoc Networks: Definition and Seminal Work

In this section, we present the seminal work of Gupta and Kumar [1] and their results on the asymptotic capacity of random wireless networks.

A. Definition of Throughput Capacity

If every node of the network can send with high probability at a rate of λ bits per second to its chosen destination, we say that the throughput λ is feasible [1]. We then define the throughput capacity by the maximum feasible throughput λ with high probability (asymptotically approaching 1).

Following [1], the *throughput capacity* of a random wireless network is said to be of order $\Theta(f(n))$ bits per second if there exist two positive constants c and $c' < \infty$ such that

$$\lim_{n\to\infty} Prob\left\{\lambda(n) = cf(n) \text{ is feasible}\right\} = 1,$$

$$\liminf_{n\to\infty} Prob\left\{\lambda(n) = c'f(n) \text{ is feasible}\right\} < 1.$$

B. Uniform Traffic Result

Following Gupta and Kumar [1], we consider a uniform model, where the nodes do not move during transmission. The nodes are independently uniformly distributed in the unit area disc. They consider an extremely simple communication model which assumes a uniform traffic pattern (i.e., source-destination pairs are chosen i.i.d). The destination for each relaying node is its closest neighbors. The following results represent the Main Result 4 in [1] and yield upper and lower bounds on the asymptotically feasible throughput:

Theorem 5.1: There exist constants c and c' such that

$$\lim_{n\to\infty} Prob\left\{\lambda(n) = \frac{cR}{\sqrt{n\log n}} \text{ is feasible}\right\} = 1,$$

and

$$\lim_{n\to\infty} Prob\left\{\lambda(n) = \frac{c'R}{\sqrt{n}} \text{ is feasible}\right\} = 0$$

R is a parameter which depends on the attenuation model, the interferences threshold and the system bandwidth.

Consequently, for the physical model where the nodes are fixed, the throughput per source-destination pair is $\Theta(\frac{1}{\sqrt{n\log n}})$. Even if this result is not particulary encouraging since the throughput goes to zero when the number of nodes increases, it represents a starting point for further analysis.

VI. Upper bounds on the Multicast Capacity

In this section, we derive the throughput capacity of an ad hoc network with multicast traffic under a hierarchical routing strategy. We will analyze two cases: the high dense multicast case and the low dense multicast.

A. The Cluster Traffic

We suppose for the routing strategy that the unit square is divided into k square clusters (cells). If we note $\bar{L}$ the average path length of a S-D pair in the unit square, we have to express the relation between $\bar{L}$ and $\bar{L_k}$ the average path length of a S-D in the clusters. If we suppose that the clusters are the result of a reduction of the unit square to a smaller one of $1/k$ area, we can translate the well known results of Gupta and Kumar to a smaller area network. On the other hand, the necessary and sufficient condition of the network connectivity in each cluster is that the transmission range for each node is $r(n/k) = \sqrt{\frac{\log(n/k)}{(n/k)}}$ [8], since we have in average n/k nodes in each cell.

B. The Upper Bound

1) Achievable traffic: Following the interference model presented above, if we consider two simultaneous successful transmissions from node X_i to node X_j and from node X_k to node X_l, we have:

$$\begin{aligned} d_{kj} &\geq (1+\Delta)d_{ij} \\ d_{kl} + d_{jl} &\geq (1+\Delta)d_{ij} \\ d_{jl} &\geq (1+\Delta)d_{ij} - d_{kl} \end{aligned}$$

and similarly,

$$\begin{aligned} d_{il} &\geq (1+\Delta)d_{kl} \\ d_{ij} + d_{jl} &\geq (1+\Delta)d_{kl} \\ d_{jl} &\geq (1+\Delta)d_{kl} - d_{ij} \end{aligned}$$

Combining the above two inequalities, we obtain

$$\begin{aligned} 2d_{jl} &\geq (1+\Delta)(d_{ij} + d_{kl}) - dkl - d_{ij} \\ d_{jl} &\geq \left(\frac{\Delta}{2}\right)(d_{ij} + d_{kl}) \end{aligned}$$

This result implies that if we place a disc around each receiver of radius $\Delta/2$ times the length of the hop, the discs must be disjoint for successful transmission under the Protocol model. Since a node transmits at W bits per second, each bit transmission time is $1/W$ seconds. During each bit transmission, the total area covered by the discs surrounding the receivers must be less than the total unit area. If we note $T(n)$ the number of simultaneous transmissions in the network, we have:

$$T(n) * \pi\left(\frac{\Delta}{2}r(n)\right)^2 \leq 1$$

Then, we have at most $\frac{4W}{\pi\Delta^2(r(n))^2}$ bps that can be transmitted by the network at any time instant.

2) Offered traffic: If every node creates traffic at rate $\lambda(n)$, the aggregate created traffic by n source nodes is $n\lambda(n)$. Each packet generated by a source must be sent to m destinations by multi-hop routing strategy. We must count the average number of hops needed to achieve the transmission to the destinations. For the 2-level hierarchical routing strategy, we have:

- Level 1: k paths to reach the cluster heads, so the same packet must be transmitted at least $k\frac{\bar{L}}{r(n)}$ times, where $\bar{L}$ is the average distance between source and destination nodes in the unit square.
- Level 2 (cluster traffic): m/k paths on average to reach the destinations in each cluster, so the same packet must be transmitted at least $\frac{m}{k}\frac{\bar{L}_k}{r(n/k)}$ times, where $\bar{L}_k$ is the average distance between source and destination nodes in the cluster of $1/k$ area. Then, since we have the same traffic in all the k clusters, the total number of hops for the second level routing step is $m\frac{\bar{L}_k}{r(n/k)}$
- The total number of hops from a source to its m destination under a 2-level hierarchical routing is $H_k(n,m)$:

$$H_k(n,m) = k\frac{\bar{L}}{r(n)} + m\frac{\bar{L}_k}{r(n/k)}$$

Finally, the offered traffic is $n\lambda(n)H_k(m,n)$.

3) The Upper Bound Expression: Offered traffic must be less than achievable traffic, so:

$$\begin{aligned} C_m(n) &= m * n\lambda(n) \\ &\leq \frac{4W}{\pi\Delta^2(r(n))^2}\frac{m}{H_k(m,n)} \end{aligned} \quad (4)$$

Let us start by the evaluation of the number of hops:

$$\begin{aligned} H_k(m,n) &= k\frac{\bar{L}}{r(n)} + m\frac{\bar{L}_k}{r(n/k)} \\ &= \frac{\bar{L}}{r(n)}\left(k + m\frac{\bar{L}_k}{\bar{L}}\frac{r(n)}{r(n/k)}\right) \end{aligned}$$

We simplify the quotient $\frac{r(n)}{r(n/k)}$ by using the minimum values of $r(n)$ and $r(n/k)$ which insure the connectivity in the entire network and in each cluster:

$$\begin{aligned} \frac{r(n)}{r(n/k)} &= \sqrt{\frac{\log n}{n}\frac{(n/k)}{\log(n/k)}} \\ &= \frac{1}{\sqrt{k}}\sqrt{\frac{\log n}{\log(n/k)}} \end{aligned}$$

So, we obtain:

$$H_k(m,n) = \frac{\bar{L}}{r(n)}\left(k + \frac{m}{k^{\frac{3}{2}}}\sqrt{\frac{\log n}{\log(n/k)}}\right)$$

Replacing this expression of $H_k(n,m)$ in (4) yields:

$$\begin{aligned} C_m(n) &\leq \frac{4W}{\pi\Delta^2}\frac{1}{r(n)^2}\frac{m}{\frac{\bar{L}}{r(n)}\left(k + \frac{m}{k^{\frac{3}{2}}}\sqrt{\frac{\log n}{\log(n/k)}}\right)} \\ &\leq \frac{4W}{\pi\Delta^2}\frac{m}{k + \frac{m}{k^{\frac{3}{2}}}\sqrt{\frac{\log n}{\log(n/k)}}}\sqrt{\frac{n}{\log n}} \end{aligned} \quad (5)$$

From the definition it follows that the upper bound on the multicast throughput capacity with a 2-level hierarchical routing is given as:

$$C_m(n) = O\left(\frac{m}{k + \frac{m}{k^{\frac{3}{2}}}\sqrt{\frac{\log n}{\log(n/k)}}}\sqrt{\frac{n}{\log n}}\right) \quad (6)$$

and the multicast gain of the routing strategy by:

$$g_k(n,m) = \frac{m}{k + \frac{m}{k^{\frac{3}{2}}}\sqrt{\frac{\log n}{\log(n/k)}}} \quad (7)$$

For the low density case, the first traffic level would be lower and we would have only k' clusters involved in the communication scheme. The multicast gain would be in this case:

$$g_{k'}(n,m) = \frac{m}{k' + \frac{m}{k^{\frac{3}{2}}}\sqrt{\frac{\log n}{\log(n/k)}}} \quad (8)$$

where $k' = k - k(1-\frac{1}{k})^m$.

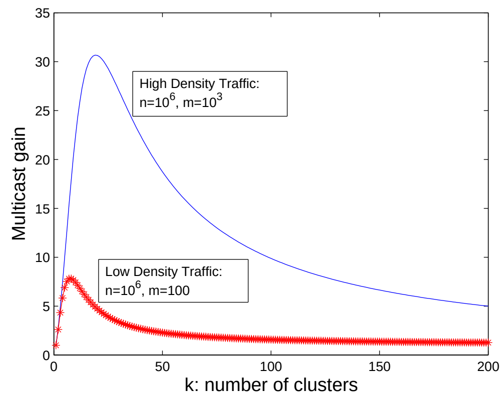

Fig. 6. Multicast Gain for High and Low density Traffics

In figure 6, we show the variation of the multicast gain as a function of the number of clusters k for the case of high traffic density, $g_k(n,m)$, and for the low density traffic case, $g_{k'}(n,m)$. It appears that the hierarchical strategy with clustering is more efficient for high density multicast traffic than for the low one. In fact, the aggregate capacity is about 30 times larger than the one obtained by a routing strategy where the same data packet is sent m times to the m destinations. Moreover, when the number of destinations is low, we can find more appropriate routing strategies [12], [13] using trees which minimize the total number of hops to reach all the destinations.

C. Multi-level Hierarchical routing

The strategy used above is a 2-level hierarchical routing, we can also explore the benefits of using more hierarchical levels in the routing strategy by dividing each cluster in k sub-clusters and so on. Let's consider a multi-level hierarchical routing strategy. Then using induction, we can prove that the multicast gain of the strategy is as follows:

$$g_{k,l}(n,m) = \sum_{i=0}^{l-2} k^{\frac{2-i}{2}}\sqrt{\frac{\log n}{\log(n/k^i)}} + \frac{m}{k^{\frac{3(l-1)}{2}}}\sqrt{\frac{\log n}{\log\left(\frac{n}{k^{l-1}}\right)}}$$

For an l-level hierarchical routing strategy, $l \geq 2$.

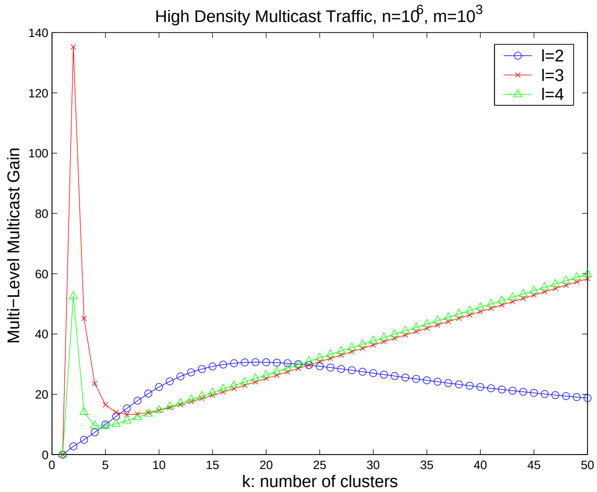

Fig. 7. Multi-level Multicast Gain for High density Traffic

Figure 7 shows the effect on the multicast gain of increasing the number of levels of hierarchy for the routing protocol. We remark that for a 3-level, we obtain the best gain with less clusters. However, when the number of clusters grows, the more the number of levels grows the more the multicast gain increases and eventually goes to infinity. A trade-off between the number of hierarchy levels and the number of clusters for a better multicast gain is needed to achieve an optimal aggregate capacity.

D. Comparison with Broadcast Traffic

For the broadcast traffic, the source sends the same data to all the the nodes in the network, so we can apply directly the result above with $m = n$:

$$C_b(n) = O\left(\frac{n}{k + \frac{n}{k^{\frac{3}{2}}}\sqrt{\frac{\log n}{\log(n/k)}}}\sqrt{\frac{n}{\log n}}\right)$$

The variation of the broadcast gain with the hierarchical strategy represented in Figure 8 shows that we can improve the aggregate capacity with clustering. However, in practice, we must find a trade-off between the capacity and the number of clusters used in the network to suit the implementation constraints.

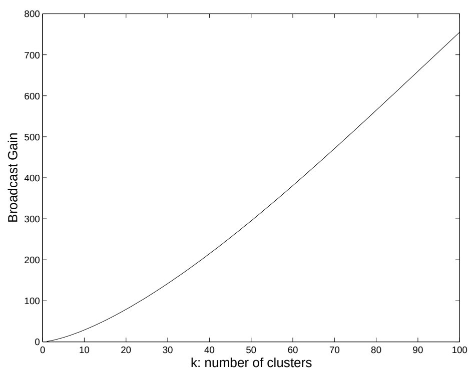

Fig. 8. 2-level Broadcast Gain, $n = 10^6$

VII. Conclusion

Following the different studies on the asymptotic performances of ad hoc networks, we present in this paper a new result on the multicast capacity of wireless ad hoc networks. By adopting a hierarchical routing strategy based on clustering, we found an upper bound on the throughput capacity for high and low density multicast traffics. We furthermore compared the gain obtained with different hierarchical levels. The gain variation with the number of clusters in the network would help to conceive efficient multicast protocols.

References

[1] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, 2002.

[2] E. Royer and C.-K. Toh, "A review of current routing protocols for ad hoc wireless networks." IEEE private communication, 1999.

[3] L. Latiff and N. Fisal, "Routing protocols in wireless mobile ad hoc network - a review," in *Proc. IEEE APCC 2003*, 2003.

[4] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," in *Proc. IEEE IFOCOM 2001*, (Anchorage, Alaska), pp. 1360–1369, 2001.

[5] A. E. Gamal *et al.*, "Throughput-delay trade-off in wireless networks," in *Proc. IEEE IFOCOM 2004*, (Hong Kong, China), 2004.

[6] L. Xie and P. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation.," *IEEE Transactions on Information Theory*, vol. 50, pp. 748–767, May 2004.

[7] R. Motwani and P. Raghavan, *Randomized Algorithms*. Cambridge university press, 1995.

[8] P. Gupta and P. Kumar, "Critical power for asymptotic connectivity in wireless networks," *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming*, pp. 547–566, 1998.

[9] O. Chaker and J. Conan, "On the throughput capacity of ad hoc wireless networks," in *Canadian Workshop On Information Theory, CWIT2005*, (Montreal, Canada), 2005.

[10] O. Chaker and J. Conan, "Bounds on throughput capacity of manets," in *14th International Conference on Computer Communications and Networks, ICCCN 2005*, (San Diego, Ca), 2005.

[11] S. Toumpis and A. Goldsmith, "Performance bounds for large wireless networks with mobile nodes and multicast traffic," in *IEEE International Workshop on Wireless Ad Hoc Networks*, (Oulu, Finland), 2004.

[12] P. Jacquet and G. Rodolakis, "Multicast scaling properties in massively dense ad hoc networks," in *11th International Conference on Parallel and Distributed Systems*, (Fukuoka, Japan), 2005.

[13] M. Gerla, C.-C. Chiang, and L. Zhang, "Tree multicast strategies in mobile, multihop wireless networks," *Mobile Networks and Applications, Baltzer Science Publishers*, vol. 4, pp. 193–207, 1999.